Well-Known Spaces Summary

Ánoq of the Sun, Hardcore Processing *

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1 Some Space Names

- $\mathbb{R}^k$:
- $\mathbb{C}^k$: Can be identified with $\mathbb{R}^{2k}$ by the following: $z = (z_1, \dots, z_k) \in \mathbb{C}^k$ is identified with $(x_1, y_1, x_2, y_2, \dots, x_k, y_k) \in \mathbb{R}^{2k}$ where for $j \in \{1, \dots, k\}$: $z_j = x_j + iy_j$ (p. 1.4 [1])
- $\mathbb{L}^k$: Either $\mathbb{R}^k$ or $\mathbb{C}^k$
- $\mathcal{F}(M, \mathbb{L})$: Vector space of *functions* $f : M \to \mathbb{L}$ (ex. 1.4 [1])
- $\mathcal{F}(\{1, 2, \dots, k\}, \mathbb{L}) \equiv \mathbb{L}^k$ (ex. 1.4 [1])
- $\mathcal{B}(M, \mathbb{L})$: Vector space of *limited functions* $f : M \to \mathbb{L}$ (ex. 1.4 [1])
- $\mathcal{B}(A)$: The set of limited functions $f : A \to A$
- $\mathcal{B}(\mathbb{N}, \mathbb{C})$: Vector space of *limited complex sequences* $z = (z_n)_{n \geq 1}$ (ex. 1.4 [1])
- $C(M, \mathbb{L})$: Continuous real or complex functions on M (p. 3.9 [1])
- $C_b(M, \mathbb{L})$: Continuous limited real or complex functions on M (p. 3.9 [1])
- $\mathrm{Hom}(E, F)$: Vector space of *linear functions* $f : E \to F$ (p. 4.6 [1])
- $\mathcal{L}(E, F) \underset{subspace}{\subseteq} \mathrm{Hom}(E, F)$: Normed vector space of *continuous linear functions* $f : E \to F$ (thm. 4.8 [1])
- $C^k([a, b], \mathbb{L})$: k times continuous differentiable functions (thm. 5.10 [1])
- $\mathcal{C}(A, B)$: The set of continuous functions $f : A \to B$
- $\mathcal{C}_0(M)$: Continous functions on M with compact support (p. A.10 [2])
- $l^0(\mathbb{N})$: The set of complex sequences $(x_n)_{n \in \mathbb{N}}$ where $\exists N \in \mathbb{N} : \forall n \geq N : x_n = 0$ (i.e. x_n is 0 from a certain point) (1.4 [2])
- $l^2(\mathbb{N})$: The set of squared summable complex sequences $(a_n)_{n \in \mathbb{N}}$. I.e. $\sum_{n=1}^{\infty} |a_n|^2 < +\infty$ (p. 1.9 [2])

- Let $B \subseteq \mathbb{R}$ be measurable. For $p = 1, 2$:
  $\mathcal{L}_p(B) \doteq \{f : B \to \mathbb{C} \mid f \text{ measurable and } \int_B |f|^p dm_1 < +\infty\}$ (p. A.9 [2])
- $\mathcal{L}_1(B)$ is also called $\mathcal{L}(B)$ (p. A.9 [2])
- $\mathcal{L}_2(B)$: The set of squared integrable complex functions on B (p. A.9 [2])
- $p = 1, 2$: $\mathcal{L}_{p,per}$: The set of complex functions on $\mathbb{R}$ with period 2π which are integrable (resp. squared integrable) on $[-\pi, \pi]$ (p. 2.4 [2])
- $\mathcal{C}_{per}$: 2π-periodic continuous complex functions on $\mathbb{R}$ (p. 2.4 [2])
- $n = 1, 2, \ldots, \infty : \mathcal{C}^n_{per}$: 2π-periodic n times continuous differentiable complex functions on $\mathbb{R}$ (p. 2.4 [2])
- $\mathcal{S}(\mathbb{R})$: The set of *Schwartz functions* (def. 6.14 [2])
- Let $B \subseteq \mathbb{R}$ be measurable. $p = 1, 2 : L_p(B) \doteq \{\widetilde{f} \mid f \in \mathcal{L}_p(B)\}$ where
  $\widetilde{f} \doteq \{g : B \to \mathbb{R} \mid g \text{ measurable}, g = f \text{ almost everywhere}\}$ (p. A.9 [2])
- $L_2([-\pi, \pi], \frac{1}{2\pi})$: equivalence classes of squared integrable complex functions on $[-\pi, \pi]$ with the inner product scaled by $\frac{1}{2\pi}$ (p. 2.1 [2])
- Results on $L_2([-\pi, \pi])$ can be generalized to results on $L_2([a, b])$ by translating and scaling $x \in [-\pi, \pi]$ into $x' \in [a, b]$ using: $x' = a + \frac{a-b}{2\pi}(x + \pi)$ (p. 2.2 [2])

References

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