

Convergence of Sequences and Series Summary

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1 Sequences

1.1 About *sequences*

- **Growing** $\stackrel{def}{=} a_1 \leq a_2 \leq \dots$ (p. 212, [1])
- **Falling** $\stackrel{def}{=} a_1 \geq a_2 \geq \dots$ (p. 212, [1])
- **Upwards limited** $\stackrel{def}{=} \exists M(\in \mathbb{R}?) : \forall n(\in I?) : a_n < M$ (p. 212, [1])
- **Downwards limited** $\stackrel{def}{=} \exists M(\in \mathbb{R}?) : \forall n(\in I?) : a_n > M$ (p. 212, [1])

1.2 Convergence of Sequences

Premises: (x_n) **sequence of real numbers**

- (x_n) *upwards limited, growing* $\Rightarrow (x_n)$ *converges* (thm 8.1, [1]) (thm 1.5, [3])
- (x_n) *downwards limited, falling* $\Rightarrow (x_n)$ *converges* (thm 8.1, [1]) (thm 1.5, [3])
- A real sequence $(x_n)_{n=1}^{\infty}$ converges towards $a \in \mathbb{R} \Leftrightarrow$
 $\liminf_{n \rightarrow \infty} x_n = \limsup_{n \rightarrow \infty} x_n = a$ (thm. 3.2 [3])

2 Series

2.1 Arithmetic Rules

- $\sum_{i=1}^{\infty} (a_i + b_i) = \sum_{i=1}^{\infty} a_i + \sum_{i=1}^{\infty} b_i,$
provided that the 2 rightmost series *converge* (p. 215, [1])
- $\sum a_i$ *converges* $\Rightarrow \sum_{i=1}^{\infty} ca_i = c \sum_{i=1}^{\infty} a_i,$
 $\sum a_i$ *diverges* $\Rightarrow \sum ca_i$ *diverges* for $c \neq 0.$ (p. 215, [1])

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2.2 Convergence

- $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$ (n'te led testen, p. 216, [1])
- If $f(x)$ is *positive, falling, continuous* on $[1, \infty[$ we have:
 $\sum_{n=1}^{\infty} f(n)$ converges $\Leftrightarrow \int_1^{\infty} f(x)dx$ converges (integral test, p. 216, [1])
- Assume $0 \leq a_n \leq b_n$ for all n . Then we have:
 $\sum b_n$ converges $\Rightarrow \sum a_n$ converges
 $\sum a_n$ diverges $\Rightarrow \sum b_n$ diverges
(comparison test, p. 217, [1])

2.3 Known Series

- $\forall a, r \in \mathbb{R} : \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ for $|r| < 1$.
 $|r| \geq 1 \Rightarrow$ *diverges* (geometric series thm. 8.2 p. 214, [1])
- The *p-series*: $p > 1 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^p}$ converges. Diverges for $p \leq 1$. Called *geometric series* for $p = 1$ (p. 217 [1])

References

- [1] Arne Hole. *Klassisk Analyse og Lineær Algebra*, Universitetsforlaget 1998.
- [2] Bergfinnur Durhuus. *Hilbert Rum med Anvendelser*, Matematisk Afdeling Københavns Universitet 1997.
- [3] Mikael Rørdam (?). *Om de Reelle Tal - Supplerende Noter til 2AN*, Maths Department University of Copenhagen 2001.